電磁學（Electromagnetism)　　Final Exam　　　　　　　　　　　　　滿分 100

[Griffiths Chs. 10 and 12]　2018/06/11,　　10:10am – 12:00am,　　　　　　教師：張存續

1. Write the equations (if possible) and explain the following terms as clear as possible.
   (a) The Lorentz gauge and the Coulomb gauge. (4%)
   (b) Gauge transformations and gauge freedom. (4%)
   (c) Phase velocity and group velocity (4%)
   (d) The two postulates of the special relativity (4%)
   (e) Invariant quantity and conserved quantity. (4%)
   (f) Lienard-Wiechert potentials. (5%)
   (g) Hidden momentum (5%)

2. (a) The transformations between two inertial systems S and $\bar{S}$ are $\bar{x}=\gamma(x-vt)$ and $\bar{t}=\gamma(t-vx/c^2)$. Show that when $\Delta t=0$, $\Delta x=\Delta\bar{x}/\gamma$; but when $\Delta\bar{t}=0$, $\Delta\bar{x}=\Delta x/\gamma$. Explain why the length relations depend on the simultaneity. (8%+2%)
   (b) Show that $(\mathbf{E}\cdot\mathbf{B})$ is relativistically invariant. (10%)
   [Hint: $\begin{matrix}\bar{E}_x=E_x, & \bar{E}_y=\gamma(E_y-vB_z), & \bar{E}_z=\gamma(E_z+vB_y)\\ \bar{B}_x=B_x, & \bar{B}_y=\gamma(B_y+\frac{v}{c^2}E_z), & \bar{B}_z=\gamma(B_z-\frac{v}{c^2}E_y)\end{matrix}$]

3. (a) $\vec{\mathscr{r}}=\mathbf{r}-\mathbf{r}'$, $\mathscr{r}=|\mathbf{r}-\mathbf{r}'|=c(t-t_r)$, and $\mathbf{v}=\mathbf{v}(t_r)$. Find $\nabla\mathscr{r}$ and $(\vec{\mathscr{r}}\cdot\nabla)\mathbf{v}$. (10%) (Note: Express your answer in terms of ∇t_r.)
   (b) Show that the retarded potentials satisfy the Lorentz gauge condition $\nabla\cdot\mathbf{A}+\mu_0\varepsilon_0\,\partial V/\partial t=0$. (10%) [Hint: the retarded potentials $V(\mathbf{r},t)=\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\mathbf{r}',t_r)}{\mathscr{r}}d\tau'$ and $\mathbf{A}(\mathbf{r},t)=\frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}',t_r)}{\mathscr{r}}d\tau'$.]

4. Suppose V=0 and $\mathbf{A}$=$A_0\cos(kx-\omega t)\,\hat{\mathbf{z}}$, where A_0, ω, and k are constants.
(a) Find **E** and **B**. (10%)
(b) Use the gauge function $\lambda=xt$ to transform the potentials V' and $\mathbf{A}'$. (10%)
(d) Find the new $\mathbf{E}'$ and $\mathbf{B}'$. Comment on the gauge freedom. (8%+2%)
   [Hint: $\mathbf{A}'=\mathbf{A}+\nabla\lambda$ and $V'=V-\partial\lambda/\partial t$]